## Appendix A - Explanation of How the Relevant APR Formula from TILA Regulation Z is Calculated Easily Using Basic Functions in Common Software

The federal Truth in Lending Act (TILA) is implemented in Regulation Z, which details the mathematical formulas for calculating the annual percentage rate (APR) in Appendix J. **The relevant Regulation Z APR formula can be calculated easily using standard industry software programs such as Microsoft Excel or Google Sheets.**

For purposes of illustration, the TILA APR equation can be reduced to the following equation via serval simplifying assumptions.[26] This TILA APR equation is explicitly designed to be used for products repaid daily, weekly, semi-monthly, monthly, and so on.

(1) $$0 = -A + \frac{P_1}{(1+i)^{t_1}} + \frac{P_2}{(1+i)^{t_2}} + \ldots + \frac{P_n}{(1+i)^{t_n}}$$ and (2) $APR = i * m$

Where: A = initial advance i = unit period interest rate (e.g. daily interest rate)
Pj = amount of the payment
Tj = number of full unit periods to the final payment
n = number of payments m = number of periods per year

**These Reg Z Formulas Can Be Calculated Using The RATE, IRR and XIRR Functions in Excel**

The following functions in Microsoft Excel or Google Sheets can be used to calculate APR consistent with the Regulation Z formula:

1. **For financing products with equal payment amounts and equal payment periods** (e.g., for loans, sales-based financing with flat sales projections or using Historical Method, etc.*)*:

   *APR = RATE (Number of payments, payment amount as a negative number, disbursed amount after fees deducted) * Number of payment periods in one year to annualize*

2. **For financing products with unequal payment amounts, and equal payment periods** (e.g., sales-based financing with projected or retrospective sales volumes that vary over the payment period, with payments every day of the week):

   *APR = IRR (select a series of cells indicating the flow of money, with the disbursed amount in the first cell, followed by cells representing the total charges in each subsequent payment period) * Number of payment periods in one year to annualize*

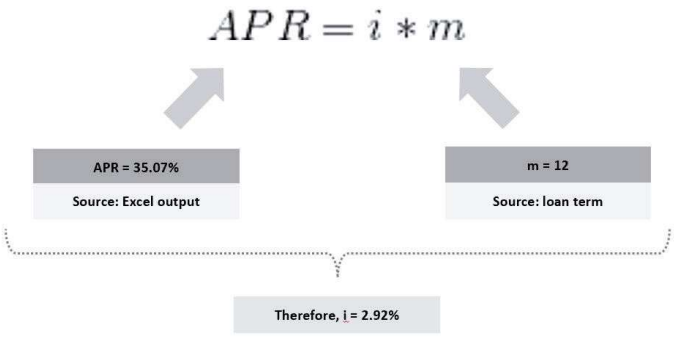
[26]Assumed here that loans have a single disbursement, and all payments occur at full unit-periods. Please see Appendix I for the full Regulation Z APR formula found in Appendix J to Part 1026(b).5.iv.

3. **For financing products with unequal payment amounts, and/or unequal payment periods** (e.g., sales-based financing with projected or retrospective sales volumes that vary over the payment period, with payments on weekdays only):

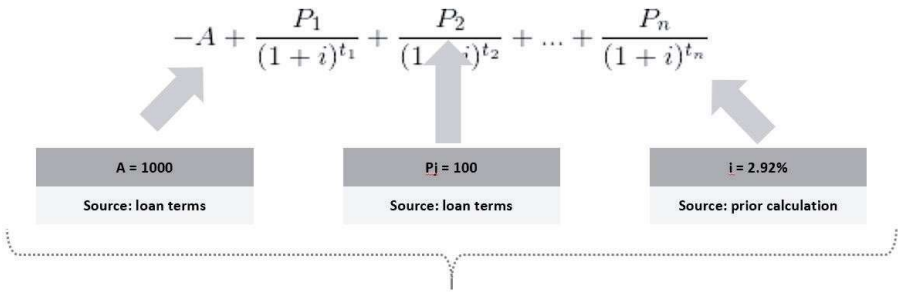
   *APR = ((XIRR (select a series of cells indicating the flow of money, starting with the disbursed amount in the first cell, and a second series of cells indicating the corresponding dates of those payments) +1)^(1/365)-1)*365*

We will use an example to show how the APR calculated from the RATE function is consistent with the TILA APR formula. To illustrate, consider a one year loan with an advance of $1,000, origination fee of $200, and 12 monthly payments of $100. To show that the APR calculated in Excel is consistent with the TILA APR formula, we will plug the APR obtained from the Excel formula described above into the TILA APR equation to show the mathematical conditions are met. The TILA APR equation will resolve to zero, proving that the Excel-derived APR correctly represents the mathematical conditions in Regulation Z.

Step 1: Using the RATE formula in Excel, we derive an APR of 35.07%. By plugging this APR and the number of periods in a year into Regulation Z equation (2), we derive a period interest rate "i" = 2.92%.




Step 2: By plugging A, P1, P2… P12 and i into equation (1), we see that equation (1) simplifies to 0 as required. This means that the Excel APR meets all the mathematical conditions dictated by the TILA APR.




$$-1000 + \frac{\$100}{(1+2.92\%)} + \frac{\$100}{(1+2.92\%)^2} + \ldots + \frac{\$100}{(1+2.92\%)^{12}}$$

$$= -1000 + 97.16 + 94.40 + \ldots + 70.77$$

$$= 0$$

Expanding beyond this example, when used correctly the RATE, IRR and XIRR functions in Excel all return an interest rate that's calculated in a manner that is mathematically consistent with that of the TILA APR.

Following are demonstrations from Microsoft Excel using each of these formulas.

**RATE Formula**

*Comparison to Reg Z*

**The following spreadsheet shows how the RATE function in Excel produces an APR that's consistent with the TILA APR methodology.**
**Use case: for loans with equal payments that occur at equal intervals**

| |
|---|
| **User Input** |
| **Calculated / Linked** |

## Section 1. Input loan terms

In Section 1, we enter the loan terms.

| | |
|---|---|
| **Initial advance** | -$1,000.00 |
| **Payment per period** | $25.00 |
| **Payment frequency** | Daily |
| **No. of total payments** | 48 |
| **Periods per year** | 365 |

| ***Payment Frequencies*** | ***Periods per Year*** |
|---|---|
| *Monthly* | *12* |
| *Semi-Monthly* | *24* |
| *Bi-Weekly* | *26* |
| *Weekly* | *52* |
| *Daily* | *365* |

**For weekday payments, see tab 'Weekday RATE'*

## Section 2. Calculate interest using RATE()

In Section 2, we use the RATE function in Excel to calculate both the interest per unit period, as well as the annual APR.

| | |
|---|---|
| **Interest rate, unit period** | 0.77% |
| **APR** | 281.10% |

## Section 3. Setting up the Reg Z Equation

In Sections 3 and 4, we verify that the APR calculated from the RATE function is in fact consistent with the TILA APR formula.

In order to do so, we will first map (in Section 3) the user inputted values about the loan to the various variables in the TILA APR formula.

$$0 = -A + \frac{P_1}{(1+i)^{1}} + \frac{P_2}{(1+i)^{2}} + \ldots + \frac{P_n}{(1+i)^{n}}$$

$$APR = i * m$$

| **Name of variable** | **Variables** | **Value** | **Source** |
|---|---|---|---|
| Initial advance | A | -$1,000.00 | User input (Section 1) |
| Payment per period | P1….Pn | $25.00 | User input (Section 1) |
| Interest per period | i | 0.77% | Calculated (Section 2) |
| Periods per year | m | 365 | Calculated (Section 1) |

## Section 4. Confirm Reg Z is Satisfied

In Section 4, we will plug the APR obtained from the Excel formula into the TILA APR formula to show the mathematical conditions are met. This is shown by taking the sum off all the values and showing it equals zero. Since cell $H$44 is zero, we have shown that the APR calculated by Excel is consistent with the TILA APR.

| **Period** | **Amount** | **Reg Z Calculation** |
|---|---|---|
| 0 | -$1,000.00 | -$1,000.00 |
| 1 | $25.00 | $24.81 |
| 2 | $25.00 | $24.62 |
| 3 | $25.00 | $24.43 |
| 4 | $25.00 | $24.24 |
| 5 | $25.00 | $24.06 |
| 6 | $25.00 | $23.88 |
| 7 | $25.00 | $23.69 |
| 8 | $25.00 | $23.51 |
| 9 | $25.00 | $23.33 |
| 10 | $25.00 | $23.15 |
| 11 | $25.00 | $22.98 |
| 12 | $25.00 | $22.80 |
| 13 | $25.00 | $22.63 |
| 14 | $25.00 | $22.45 |
| 15 | $25.00 | $22.28 |
| 16 | $25.00 | $22.11 |
| 17 | $25.00 | $21.94 |
| 18 | $25.00 | $21.78 |
| 19 | $25.00 | $21.61 |
| 20 | $25.00 | $21.44 |
| 21 | $25.00 | $21.28 |
| 22 | $25.00 | $21.12 |
| 23 | $25.00 | $20.96 |
| 24 | $25.00 | $20.80 |
| 25 | $25.00 | $20.64 |
| 26 | $25.00 | $20.48 |
| 27 | $25.00 | $20.32 |
| 28 | $25.00 | $20.17 |
| 29 | $25.00 | $20.01 |
| 30 | $25.00 | $19.86 |
| 31 | $25.00 | $19.71 |
| 32 | $25.00 | $19.56 |

| | |
|---|---|
| **Sum** | 0.00 |
| **Is sum zero (nearest cent)** | Yes |
| **Satisfies Reg Z** | Yes |

*NOTE: because this table ends at 50 rows, it will not calculate in full for examples with more than 49 payments. To calculate a protect with more payments, simply extend te number of rows in this section.*

| | | | | |
|---|---|---|---|---|
| | 33 | $25.00 | $19.41 | |
| | 34 | $25.00 | $19.26 | |
| | 35 | $25.00 | $19.11 | |
| | 36 | $25.00 | $18.97 | |
| | 37 | $25.00 | $18.82 | |
| | 38 | $25.00 | $18.68 | |
| | 39 | $25.00 | $18.54 | |
| | 40 | $25.00 | $18.39 | |
| | 41 | $25.00 | $18.25 | |
| | 42 | $25.00 | $18.11 | |
| | 43 | $25.00 | $17.97 | |
| | 44 | $25.00 | $17.84 | |
| | 45 | $25.00 | $17.70 | |
| | 46 | $25.00 | $17.57 | |
| | 47 | $25.00 | $17.43 | |
| | 48 | $25.00 | $17.30 | |
| | 49 | $0.00 | $0.00 | |
| | 50 | $0.00 | $0.00 | |

**IRR Formula**

*Comparison to Reg Z*

**The following spreadsheet shows how the IRR function in Excel produces an APR that's consistent with the TILA APR methodology.**
**For ease of illustration, we have limited the Excel template for loans with no more than 52 periods. In production, Excel can handle loans with more than 52 periods.**
**Use case: for loans with non-equal payments that occur at equal intervals.**

| **User Input** |
|---|
| **Calculated / Linked** |

## Section 1. Input loan terms

In Section 1, we enter the terms of the loan.

| **Loan Terms** | | |
|---|---|---|
| Initial advance | -$1,000.00 | *(Entered as a negative number)* |
| Period 1 payment | $90.00 | |
| Period 2 payment | $90.00 | |
| Period 3 payment | $150.00 | |
| Period 4 payment | $0.00 | |
| Period 5 payment | $0.00 | |
| Period 6 payment | $80.00 | |
| Period 7 payment | $50.00 | |
| Period 8 payment | $60.00 | |
| Period 9 payment | $140.00 | |
| Period 10 payment | $100.00 | |
| Period 11 payment | $150.00 | |
| Period 12 payment | $110.00 | |
| Period 13 payment | $0.00 | |
| Period 14 payment | $0.00 | |
| Period 15 payment | $0.00 | |
| Period 16 payment | $0.00 | |
| Period 17 payment | $0.00 | |
| Period 18 payment | $0.00 | |
| Period 19 payment | $0.00 | |
| Period 20 payment | $0.00 | |
| Period 21 payment | $0.00 | |
| Period 22 payment | $0.00 | |
| Period 23 payment | $0.00 | |
| Period 24 payment | $0.00 | |
| Period 25 payment | $0.00 | |
| Period 26 payment | $0.00 | |
| Period 27 payment | $0.00 | |
| Period 28 payment | $0.00 | |
| Period 29 payment | $0.00 | |
| Period 30 payment | $0.00 | |
| Period 31 payment | $0.00 | |
| Period 32 payment | $0.00 | |
| Period 33 payment | $0.00 | |
| Period 34 payment | $0.00 | |
| Period 35 payment | $0.00 | |
| Period 36 payment | $0.00 | |
| Period 37 payment | $0.00 | |
| Period 38 payment | $0.00 | |
| Period 39 payment | $0.00 | |
| Period 40 payment | $0.00 | |
| Period 41 payment | $0.00 | |
| Period 42 payment | $0.00 | |
| Period 43 payment | $0.00 | |
| Period 44 payment | $0.00 | |
| Period 45 payment | $0.00 | |
| Period 46 payment | $0.00 | |
| Period 47 payment | $0.00 | |
| Period 48 payment | $0.00 | |
| Period 49 payment | $0.00 | |
| Period 50 payment | $0.00 | |
| Period 51 payment | $0.00 | |
| Period 52 payment | $0.00 | |
| Payment frequency | Daily | |
| No. of total payments | 10 | |
| Periods per year | 365 | |

| ***Payment Frequencies*** | ***Periods per Year*** |
|---|---|
| *Monthly* | *12* |
| *Semi-Monthly* | *24* |
| *Bi-Weekly* | *26* |
| *Weekly* | *52* |
| *Daily* | *365* |

**For equal weekday payments, please see tab 'Weekday RATE'*

## Section 2. Calculate interest using IRR()

In Section 2, we use the IRR function in Excel to calculate both the interest per unit period, as well as the annual APR.

| | |
|---|---|
| **Interest rate, unit period** | 0.28% |
| **APR** | 101.98% |

## Section 3. Setting up the Reg Z Equation

In Sections 3 and 4, we want to verify that the APR calculated from the IRR function is in fact consistent with the TILA APR formula.

In order to do so, we will first map (in Section 3) the user inputted values about the loan to the various variables in the TILA APR formula.

$$0 = -A + \frac{P_1}{(1+i)^{t_1}} + \frac{P_2}{(1+i)^{t_2}} + \ldots + \frac{P_n}{(1+i)^{t_n}}$$

$$APR = i * m$$

| **Name of variable** | **Variables** | **Value** | **Source** |
|---|---|---|---|
| Initial advance | A | -$1,000.00 | User input (Section 1) |
| Period 1 payment | P1 | $90.00 | User input (Section 1) |
| Period 2 payment | P2 | $90.00 | User input (Section 1) |
| Period 3 payment | P3 | $150.00 | User input (Section 1) |
| Period 4 payment | P4 | $0.00 | User input (Section 1) |
| Period 5 payment | P5 | $0.00 | User input (Section 1) |
| Period 6 payment | P6 | $80.00 | User input (Section 1) |
| Period 7 payment | P7 | $50.00 | User input (Section 1) |
| Period 8 payment | P8 | $60.00 | User input (Section 1) |
| Period 9 payment | P9 | $140.00 | User input (Section 1) |
| Period 10 payment | P10 | $100.00 | User input (Section 1) |
| Period 11 payment | P11 | $150.00 | User input (Section 1) |
| Period 12 payment | P12 | $110.00 | User input (Section 1) |
| Period 13 payment | P13 | $0.00 | User input (Section 1) |
| Period 14 payment | P14 | $0.00 | User input (Section 1) |

| | | | | |
|---|---|---|---|---|
| | Period 15 payment | P15 | $0.00 | User input (Section 1) |
| | Period 16 payment | P16 | $0.00 | User input (Section 1) |
| | Period 17 payment | P17 | $0.00 | User input (Section 1) |
| | Period 18 payment | P18 | $0.00 | User input (Section 1) |
| | Period 19 payment | P19 | $0.00 | User input (Section 1) |
| | Period 20 payment | P20 | $0.00 | User input (Section 1) |
| | Period 21 payment | P21 | $0.00 | User input (Section 1) |
| | Period 22 payment | P22 | $0.00 | User input (Section 1) |
| | Period 23 payment | P23 | $0.00 | User input (Section 1) |
| | Period 24 payment | P24 | $0.00 | User input (Section 1) |
| | Period 25 payment | P25 | $0.00 | User input (Section 1) |
| | Period 26 payment | P26 | $0.00 | User input (Section 1) |
| | Period 27 payment | P27 | $0.00 | User input (Section 1) |
| | Period 28 payment | P28 | $0.00 | User input (Section 1) |
| | Period 29 payment | P29 | $0.00 | User input (Section 1) |
| | Period 30 payment | P30 | $0.00 | User input (Section 1) |
| | Period 31 payment | P31 | $0.00 | User input (Section 1) |
| | Period 32 payment | P32 | $0.00 | User input (Section 1) |
| | Period 33 payment | P33 | $0.00 | User input (Section 1) |
| | Period 34 payment | P34 | $0.00 | User input (Section 1) |
| | Period 35 payment | P35 | $0.00 | User input (Section 1) |
| | Period 36 payment | P36 | $0.00 | User input (Section 1) |
| | Period 37 payment | P37 | $0.00 | User input (Section 1) |
| | Period 38 payment | P38 | $0.00 | User input (Section 1) |
| | Period 39 payment | P39 | $0.00 | User input (Section 1) |
| | Period 40 payment | P40 | $0.00 | User input (Section 1) |
| | Period 41 payment | P41 | $0.00 | User input (Section 1) |
| | Period 42 payment | P42 | $0.00 | User input (Section 1) |
| | Period 43 payment | P43 | $0.00 | User input (Section 1) |
| | Period 44 payment | P44 | $0.00 | User input (Section 1) |
| | Period 45 payment | P45 | $0.00 | User input (Section 1) |
| | Period 46 payment | P46 | $0.00 | User input (Section 1) |
| | Period 47 payment | P47 | $0.00 | User input (Section 1) |
| | Period 48 payment | P48 | $0.00 | User input (Section 1) |
| | Period 49 payment | P49 | $0.00 | User input (Section 1) |
| | Period 50 payment | P50 | $0.00 | User input (Section 1) |
| | Period 51 payment | P51 | $0.00 | User input (Section 1) |
| | Period 52 payment | P52 | $0.00 | User input (Section 1) |
| | Interest per period | i | 0.28% | Calculated (Section 2) |
| | Periods per year | m | 365 | Calculated (Section 1) |

**Section 4. Confirm Reg Z is Satisfied**

In Section 4, we will plug the APR obtained from the Excel formula into the TILA APR formula to show the mathematical conditions are met. This is shown by taking the sum off all the values and showing it equals zero. Since cell $H$148 is zero, we have shown that the APR calculated by Excel is consistent with the TILA APR.

| Period | Amount | Reg Z Calculation |
|---|---|---|
| 0 | -$1,000.00 | -$1,000.00 |
| 1 | $90.00 | $89.75 |
| 2 | $90.00 | $89.50 |
| 3 | $150.00 | $148.75 |
| 4 | $0.00 | $0.00 |
| 5 | $0.00 | $0.00 |
| 6 | $80.00 | $78.67 |
| 7 | $50.00 | $49.03 |
| 8 | $60.00 | $58.68 |
| 9 | $140.00 | $136.53 |
| 10 | $100.00 | $97.25 |
| 11 | $150.00 | $145.47 |
| 12 | $110.00 | $106.38 |
| 13 | $0.00 | $0.00 |
| 14 | $0.00 | $0.00 |
| 15 | $0.00 | $0.00 |
| 16 | $0.00 | $0.00 |
| 17 | $0.00 | $0.00 |
| 18 | $0.00 | $0.00 |
| 19 | $0.00 | $0.00 |
| 20 | $0.00 | $0.00 |
| 21 | $0.00 | $0.00 |
| 22 | $0.00 | $0.00 |
| 23 | $0.00 | $0.00 |
| 24 | $0.00 | $0.00 |
| 25 | $0.00 | $0.00 |
| 26 | $0.00 | $0.00 |
| 27 | $0.00 | $0.00 |
| 28 | $0.00 | $0.00 |
| 29 | $0.00 | $0.00 |
| 30 | $0.00 | $0.00 |
| 31 | $0.00 | $0.00 |
| 32 | $0.00 | $0.00 |
| 33 | $0.00 | $0.00 |
| 34 | $0.00 | $0.00 |
| 35 | $0.00 | $0.00 |
| 36 | $0.00 | $0.00 |
| 37 | $0.00 | $0.00 |
| 38 | $0.00 | $0.00 |
| 39 | $0.00 | $0.00 |
| 40 | $0.00 | $0.00 |
| 41 | $0.00 | $0.00 |
| 42 | $0.00 | $0.00 |
| 43 | $0.00 | $0.00 |
| 44 | $0.00 | $0.00 |
| 45 | $0.00 | $0.00 |
| 46 | $0.00 | $0.00 |
| 47 | $0.00 | $0.00 |
| 48 | $0.00 | $0.00 |
| 49 | $0.00 | $0.00 |
| 50 | $0.00 | $0.00 |
| 51 | $0.00 | $0.00 |
| 52 | $0.00 | $0.00 |

| | |
|---|---|
| **Sum** | 0.00 |
| **Is sum zero (nearest cent)** | Yes |
| **Satisfies Reg Z** | Yes |